Insights project

E29 - Distinct powers

Consider all integer combinations of a^b for $2 \le a \le 5$ and $2 \le b \le 5$:

$$\begin{array}{llll} 2^2 = 4 & ,2^3 = 8 & ,2^4 = 16 & ,2^5 = 32 \\ 3^2 = 9 & ,3^3 = 27 & ,3^4 = 81 & ,3^5 = 243 \\ 4^2 = 16 & ,4^3 = 64 & ,4^4 = 256 & ,4^5 = 1024 \\ 5^2 = 25 & ,5^3 = 125 & ,5^4 = 625 & ,5^5 = 3125 \end{array}$$

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 15 distinct terms:

4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125

How many distinct terms are in the sequence generated by a^b for $2 \le a \le 30$ and $2 \le b \le 5$?

Guides (you will need to submit these for full credit):

1. If we are considering a^b for $2 \le a \le 8$ and $2 \le b \le 5$:
   a. Which values of 'a' are candidates to create duplicates when raised to the power of 'b' ?
   b. What are the duplicates, and how many distinct terms there are in the overall sequence?

2. If we are considering a^b for $2 \le a \le 9$ and $2 \le b \le 5$:
   a. Which values of 'a' are candidates to create duplicates when raised to the power of 'b' ?
   b. What are the duplicates, and how many distinct terms there are in the overall sequence?

Now, generalize your result to the case given.

➔ Keep in mind: You need to show a detailed explanation of your solution.

* About the name: This problem is based on Euler Project problem number 29.

=== End ====