Name:___________________________ Date:_________

# Class worksheet: Alg2H
## Powers and Roots (III): Rational numbers as exponents
(book chapter 7, page 310 to 317)

### Roots and exponents

Theorem 7-6.

$$\boxed{\sqrt[k]{a^m} = \left(\sqrt[k]{a}\right)^m}$$

$a \geq 0$, $k, m$ integer, $k > 0$

similar to $(a^m)^k = (a^k)^m$

$$\left(\sqrt[3]{2}\right)^6 = \big(\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}}_{2}\cdot\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}}_{2}\big) = \underline{\underline{4}}$$

$$\left(\sqrt[3]{2^6}\right) = \left(\sqrt[3]{64}\right) = \underline{\underline{4}}$$

### Rational numbers as exponents

$x^{\frac{1}{2}} = ?$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}} = x^1 = x$ } Let's <u>define</u> $x^{\frac{1}{2}} = \sqrt{x}$.

$\sqrt{x}\cdot\sqrt{x} = x$

**<u>Definition:</u>** $\boxed{a^{\frac{1}{k}} = \sqrt[k]{a}}$ $a \geq 0$, $k \in \mathbb{N}$

when working w/Rational exponents, we will assume base is non-negative

The <u>non</u>-negative.

$a^{\frac{2}{3}} = ?$

$(a^{\frac{1}{3}})^2 = (\sqrt[3]{a})^2$

$(a^2)^{\frac{1}{3}} = \sqrt[3]{a^2}$

<u>Definition</u>

$$\boxed{a^{\frac{m}{k}} = \sqrt[k]{a^m} = \left(\sqrt[k]{a}\right)^m}$$

## Rational numbers as exponents (Cont.)

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$

$\sqrt[4]{x^2y^3} = \left(x^2y^3\right)^{\frac{1}{4}} = x^{\frac{2}{4}} \cdot y^{\frac{3}{4}} = \boxed{x^{\frac{1}{2}} \cdot y^{\frac{3}{4}}}$

## Negative rational exponents

Definition

$$\boxed{a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}}$$

$\frac{m}{n}$ rational

$a \geq 0$

$a^{\frac{m}{n}} \cdot a^{-\frac{m}{n}} = 1 \leftarrow$ reciprocals

$4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt{4}} = \boxed{\frac{1}{2}}$

$2^{\frac{4}{5}} \cdot 2^{\frac{6}{5}} = 2^{\frac{10}{5}} = 4$

$\frac{5^{\frac{4}{3}}}{5^{\frac{1}{3}}} = 5$

$\left(9^{\frac{4}{3}}\right)^{\frac{3}{8}} = 9^{\frac{1}{2}} = 3$