Name:______________________________ Date:__________

Class worksheet: Alg2H
Complex Numbers (I): Definition, addition and multiplication
(book chapter 7-7, page 321 to 329)

Define Negative numbers do not have real square roots. Yet, we want to keep manipulate these. so we invent/define

$$i^2 = -1 \quad \text{or} \quad i = \sqrt{-1}$$

Roots of negative numbers:

$$\sqrt{-5} = \sqrt{-1}\cdot\sqrt{5} = \boxed{i\sqrt{5}} \quad \text{or} \quad \boxed{\sqrt{5}\,i}$$

$\sqrt{-1\cdot 5}$

$$\sqrt{-36} = i\sqrt{36} = \boxed{6i}$$

Multiples of i
(Powers)

$i^1 = i$

$i^2 = -1$

$i^3 = i^2\cdot i = -i$

$i^4 = i^2\cdot i^2 = (-1)(-1) = 1$

$i^5 = i^4\cdot i = i$

$i^6 = i^4\cdot i^2 = -1$

$i^{12} = 1$

$i^{13} = i$

$\vdots$

$i^{18} = i^2\, i^{16} = -1$

$\vdots$

$i^{83} = i^{80}\cdot i^3 = -i$

Very common pitfall

$$\sqrt{-3}\cdot\sqrt{-12} \to \text{what we learned today } i\sqrt{3}\cdot i\cdot\sqrt{12} = i^2\cdot\sqrt{36} = \boxed{-6}$$

$\to$ Wrong way:

$$\sqrt{-3}\cdot\sqrt{-12} \cancel{=} \sqrt{-3\cdot -12} = \sqrt{36} = 6$$

Complex numbers

a - real number

b - real number

$b\cdot i \to$ Imaginary number

$a + bi \to$ Complex number

$$\underbrace{\underbrace{2}_{\text{Real part}} + \underbrace{3i}_{\text{Imaginery part}}}_{\text{Complex Number}}$$

Adding and multiplying

Adding

1. $-7i + 10i = 3i$

2. $(3+2i) + (5-3i) = 8 - i$

3. $(4+3i) - (2+3i) = 2$ ← real number

multiplying

$3 \cdot (2+3i) = 6 + 9i$

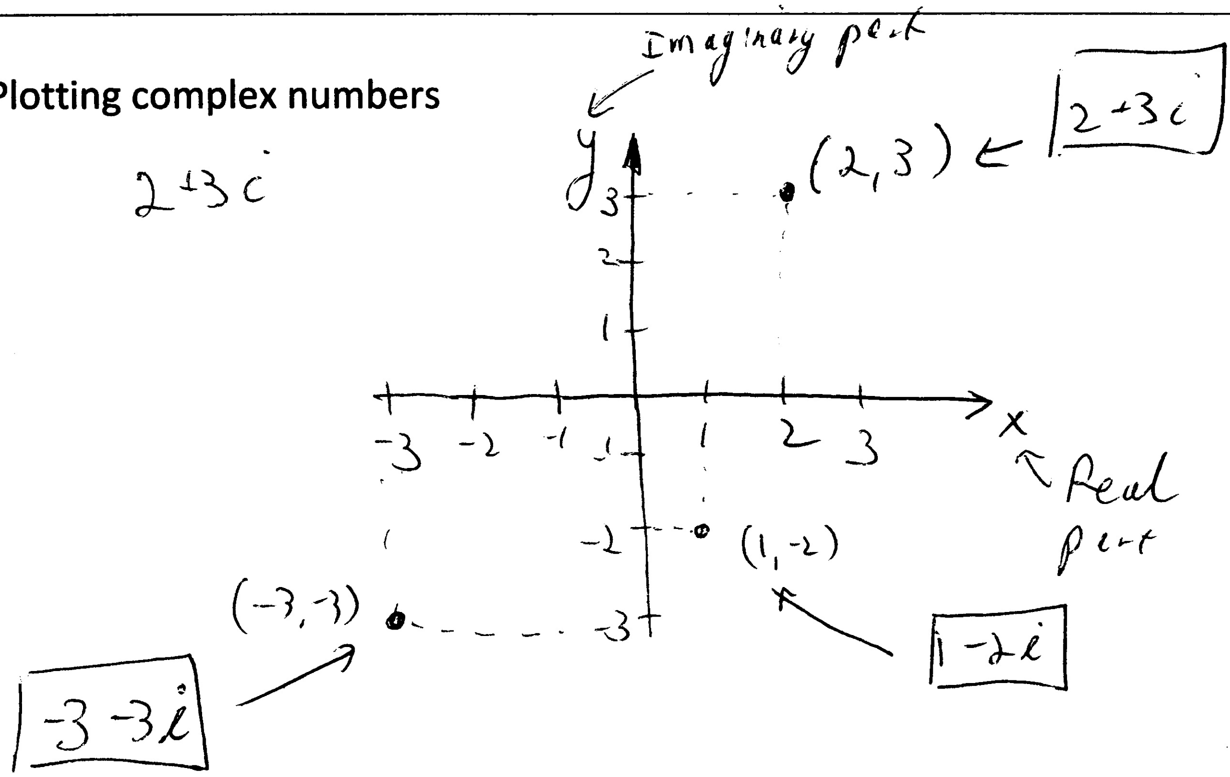
$2i(2+3i) = 4i + 6i^2 = 4i - 6 = \boxed{-6+4i}$

$(2+3i)(1+4i) = 2 \cdot 1 + 2 \cdot 4i + 3i \cdot 1 + 3 \cdot 4 \cdot i^2$

$= 2 + 8i + 3i - 12 = \boxed{-10+11i}$

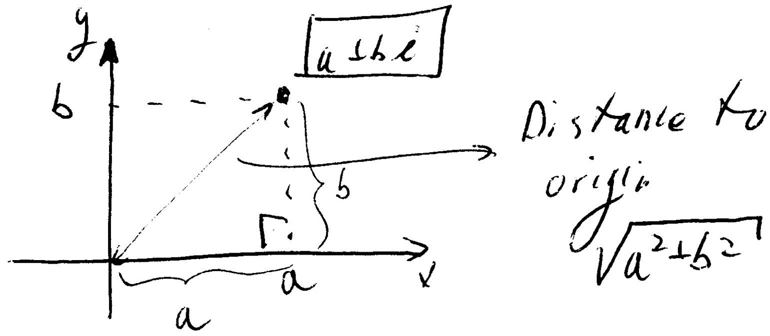
Plotting complex numbers

$2+3i$



General

$a+bi$



Absolute value

Distance to origin. For complex numbers, we <u>define</u> it as

$$\boxed{|a+bi| = \sqrt{a^2+b^2}}$$