Name:_____ Date:_____

Class/Home worksheet: Alg2H
Polynomials + Factoring: Intro (book chapter 5, P. 206)

Definition: **A Polynomial is** any expression of the form

$$a_n x^n + a_{n-1}x^{n-1} + \ \ldots\ + a_1 x^1 + a_0$$

where n is a non-negative integer and the coefficients are real numbers.

Polynomials vocabulary: $2x^2 - 3x + 4$

1. Has 3 **terms** : $2x^2$, $-3x$, 4
2. **Coefficients:** 2 , -3 , 4
3. **Degree of a term:** 2 , 1 , 0
4. **Degree of polynomial** : The highest degree of its terms : 2

Polynomial function

$$P(x) = 4x^3 - 2x^2 + 3x - 4$$

1. Has 4 **terms** : $4x^3$, $-2x^2$, $+3x$, -4
2. **Coefficients:** $+4$, -2 , $+3$, -4
3. **Degree of a term:** 3 , 2 , 1 , 0
4. **Degree of polynomial** : The highest degree of its terms : 3
5. **P(0) = P (x=0) =** $4\cdot 0^3 - 2\cdot 0^2 + 3\cdot 0 - 4 = \boxed{-4}$
6. **P(2) =** $4\cdot 2^3 - 2\cdot 2^2 + 3\cdot 2 - 4 = 32 - 8 + 6 - 4 = \boxed{26}$

Problems denoted with ** mark are taken from Exeter Phillips Academy (NH) math curriculum.

Collecting **like terms**: Two terms have the SAME variable raised to the SAME power → Called similar terms. (in the book, p 207)

Collect like terms:

$3x + 2x^2 + 4x + 5x^2 =$

$7x + 7x^2$

Collect like terms:

$4x^2y + xy + 5xy + 3x^2y =$

$7x^2y + 6xy$

Collect like terms:

$1 + 2x^2y + 3xy + 4yx^2 + 5xy =$

$1 + 6x^2y + 8xy$

Adding polynomials:

$(2x^3 + 3x) + (6x^3 + 4x + 1) =$

$8x^3 + 7x + 1$

Subtracting polynomials:

$(5x + 7xy + 6x^2y) - (2x + 3xy + 4x^2y) =$

$5x + 7xy + 6x^2y - 2x - 3xy - 4x^2y =$

$= 3x + 4xy + 2x^2y$

Subtracting polynomials:

$(4xy^2 - 6x^2y^2 + 5x^3y^2) - (2xy^2 + 4x^2y^2 - 8x^3y^2) =$

$2xy^2 - 10x^2y^2 + 13x^3y^2$

Multiply polynomials (5-3 in the book, p. 214) FOIL

$(3x + 2y) \cdot (5x + y) =$ $3x \cdot 5x + 3x \cdot y + 2y \cdot 5x + 2y \cdot y =$

$= 15x^2 + 3xy + 10xy + 2y^2 = \boxed{15x^2 + 13xy + 2y^2}$

Multiply:

$(2x + 20) \cdot (3y - 20) =$

$6xy - 40x + 60y - 400$

Multiply:

$(3xy + 2x) \cdot (x^2 + 2xy^2) =$

$3x^3y + 6x^2y^3 + 2x^3 + 4x^2y^2$

Square of Binomials (P. 215)

$(A+B)^2 =$

$(A+B)(A+B) = \boxed{A^2+2AB+B^2}$

Simplify:

$(x+3)^2 =$

$x^2+2\cdot 3\cdot x+(3)^2 =$

$= \boxed{x^2+6x+9}$

Simplify:

$(x-3)^2 =$

$x^2+2\cdot(-3)\cdot x+(-3)^2 =$

$= \boxed{x^2-6x+9}$

Simplify:

$(y-5)^2 =$

$y^2-10y+25$

Simplify:

$(4x-5y)^2 =$

$16x^2-40xy+25y^2$

Products of Sums and Differences (P. 216)

$(A+B)\cdot(A-B) =$ $A^2-AB+BA-B^2 = \boxed{A^2-B^2}$

Simplify:

$$(x+4)(x-4) =$$

$x^2 - 16$

Simplify:

$$(3xy-4)(3xy+4) =$$

$(3xy)^2 - (4)^2 =$

$= \boxed{9x^2y^2 - 16}$

Simplify (Challenging):

$$(2x+1+y)\cdot(2x+1-y) =$$

$[(2x+1)+y]\cdot[(2x+1)-y] = (2x+1)^2 - y^2 =$

$= \boxed{4x^2+4x+1-y^2}$

Cubing polynomials

$(A+B)^3 =$ $(A+B)(A^2+2AB+B^2) =$

$= A^3 + 2A^2B + AB^2 + BA^2 + 2AB^2 + B^3$

$= \boxed{A^3 + 3A^2B + 3AB^2 + B^3}$

Simplify:

$(x+2)^3 =$ $x^3 + 3x^2\cdot 2 + 3x\cdot 2^2 + 2^3 =$

$= \boxed{x^3 + 6x^2 + 12x + 8}$

Simplify:

$(x-2)^3 =$ $\boxed{x^3 - 6x^2 + 12x - 8}$

Questions from the book, pages 217-218.
Simplify.

(5) $(a-b)(a^2+ab+b^2)$

$a^3+a^2b+ab^2-ba^2-ab^2-b^3$

$=a^3-b^3$

(6) $(t+1)(t^2-t+1)$

$t^3-t^2+t+t^2-t+1=$

t^3+1

(15) $(2x^2-3y)^2$

$4x^4-12x^2y+9y^2$

(19) $(3x-2y)(3x+2y)$

$9x^2-4y^2$

(39) Challenging:

$(x^a+y^b)(x^a-y^b)(x^{2a}+y^{2b})$

$(x^{2a}-y^{2b})(x^{2a}+y^{2b})$

$=x^{4a}-y^{4b}$

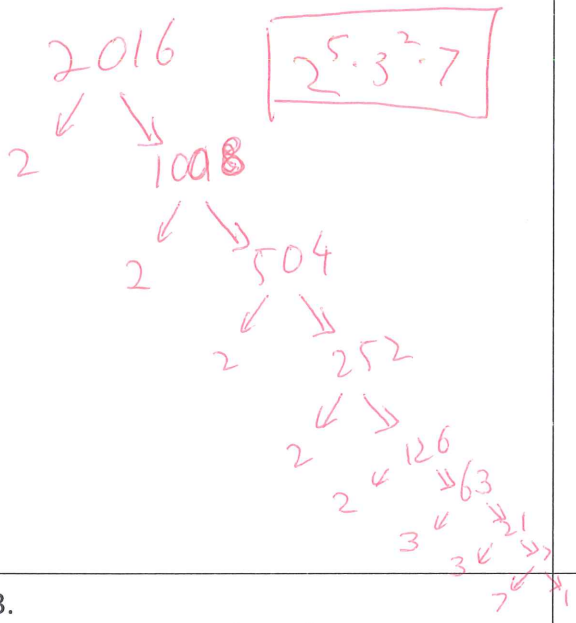
(40) Challenging:

$(x^{a-b})^{a+b}$

$x^{(a-b)(a+b)}=x^{a^2-b^2}$

Factoring (preparation!)

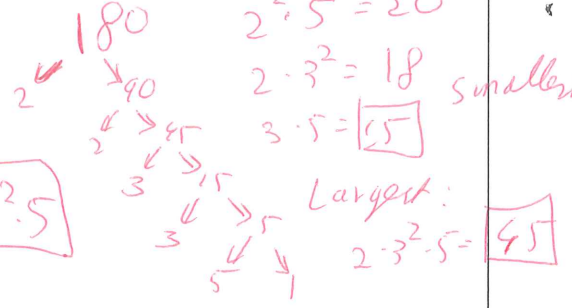
1. Factor the number 2016 (last year!)



$2^5 \cdot 3^2 \cdot 7$

2. (Book1 628**) There are a few positive integers, that when are used to divide 192 leave a remainder of 12. Find the smallest and the largest of those integers.

need > 12



$2^2 \cdot 5 = 20$

$2 \cdot 3^2 = 18$ smallest

$3 \cdot 5 = 15$

$2^2 \cdot 3^2 \cdot 5$

Largest:

$2 \cdot 3^2 \cdot 5 = 45$

3. $\frac{1}{12} + \frac{1}{18} =$

$\frac{3}{36} + \frac{2}{36} = \frac{5}{36}$

4. $\frac{1}{12} + \frac{1}{18} + \frac{1}{24} =$

$\frac{6+4+3}{72} = \frac{13}{72}$

5. One more to ponder:

An old Sheikh is on his death-bed, calls his three sons, and says: "I am about to die, and I want to make sure I split my herd of camels properly among you. To my eldest, I give half of the camels. For the second, since he also has fields of his own, I will leave one third of my herd. To the youngest, who by now has his own startup and is very strong in math, I will leave only $\frac{2}{9}$ of my herd." With that, the Sheikh takes his last breath. The three sons go out, and find out the herd contains 17 camels. How can they fulfill their dad's last wish, WITHOUT splitting any camel? How many camels does each get?