Homework sheet: Alg2H
Systems of equations: Mixtures and more word problems

has ✓

1. A chemist ~~solution A~~ "solution A" that is 20% acid, and a "solution B" that is 60% acid. How many litters of each should be mixed to get 10 litters of a solution that is ___% acid?

   a. **To get 10 litters of a solution that is 20% acid?**

   Answer: __10__ litters of solution A ; __0__ litters of solution ~~A~~ B
   ==

   b. **To get 10 litters of a solution that is 60% acid?**

   Answer: __0__ litters of solution A ; __10__ litters of solution ~~A~~ B
   ==

   c. **To get 10 litters of a solution that is 40% acid?**

   $A \to x$
   $B \to y$ $\quad x + y = 10$

   $$\frac{x}{10} \cdot 20\% + \frac{y}{10} \cdot 60\% = 40\% \Rightarrow \boxed{x = y = 5}$$

   Answer: __5__ litters of solution A ; __5__ litters of solution ~~A~~ B
   ==

   d. **To get 10 litters of a solution that is 50% acid?**

   $$\frac{x}{10} * 20\% + \frac{y}{10} \cdot 60\% = 50\% \to 2x + 6y = 50 \Big\} \begin{array}{l} 2x + 6(10-x) = 50 \\ -4x = -10 \\ x = 2.5 \\ y = 7.5 \end{array}$$
   $$x + y = 10 \to y = 10 - x$$

   Answer: __2.5__ litters of solution A ; __7.5__ litters of solution ~~A~~ B
   ==

   e. **To get 10 litters of a solution that is 45% acid?**

   $$\left. \begin{array}{l} 2x + 6y = 45 \\ y = 10 - x \end{array} \right\} \to \begin{array}{l} -4x = -15 \\ x = \frac{15}{4} = 3\frac{3}{4} \end{array}$$

   Answer: __$3\frac{3}{4}$__ litters of solution A ; __$6\frac{1}{4}$__ litters of solution ~~A~~ B

Problems denoted with ** mark are taken from Exeter Phillips Academy (NH) math curriculum.

2. (From course book: Page 171, Question 5)

Soybean meal is 16% protein and corn meal is 9% protein. How many pounds of each should be mixed together to get a 350-Lb mixture that is 12% protein?

Soybean $-X$
Corn $-y$

$X + y = 350$

$\frac{X}{350} \cdot 16\% + \frac{y}{350} \cdot 9\% = 12\%$

$\begin{cases} 16X + 9y = 12 \cdot 350 \\ X + y = 350 \rightarrow y = 350 - X \end{cases}$

$16X - 9X = (12 - 9) 350 \rightarrow \boxed{X = \frac{3 \cdot 350}{7} = 150}$

$\boxed{y = 200}$

3. (From course book: Page 17~~1~~2, Question 21)

Carlos is 8 years older than his sister Maria. Four years ago Maria was two thirds as old as Carlos. How old are they now?

Carlos X
Maria y

4 years ago

$\left. \begin{array}{c} X = y + 8 \\ y = \frac{2}{3} X \end{array} \right\}$

$X = \frac{2}{3} X + 8$
$\frac{1}{3} X = 8$
$X = 24 \rightarrow$ Today $\boxed{\begin{array}{c} X = 28 \\ y = 20 \end{array}}$
$y = 16$

4. (From course book: Page 17~~1~~2, Question 16)

Two planes travel toward each other from cities that are 780 Miles apart, at speeds of 190mph and 200mph (small slow planes). They started at the same time. In how many hours will they meet?

$190 \cdot t + 200 \cdot t = 780$

$\boxed{t = 2 \, hrs}$