

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Class worksheet: Alg2H  
Arithmetic sequence and series  
(book chapter 14)

Warm up:  $0, 5, 10, 15, \dots$   $d=5$   
 $0, \frac{1}{16}, \frac{1}{8}, \frac{3}{16}, \frac{1}{4}, \dots$   $d=\frac{1}{16}$   
 $1, -2, 4, -8, 16, \dots$   $r=-2$

Definition: Arithmetic sequence Common difference,  $d$   
 $3, 6, 9, 12, \dots$   
 $7, 9, 11, 13, 15, \dots$

Recursive formula Explicit formula

$1, 3, 5, 7, \dots$   
 $d=2$

$a_{n+1} = a_n + d$

$a_n = a_1 + (n-1)d$

$a_{n+1} = a_n + 2$        $a_n = 1 + (n-1) \cdot 2$   
 $= 2n - 1$

$a_1 \quad a_2 \quad a_3 \quad a_4$   
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

Find the n'th element:

$10, 8, 6, 4, \dots \Rightarrow d = -2$

$$a_n = a_1 + (n-1) \cdot d$$

$$\boxed{a_{20}} = 10 + (20-1) \cdot (-2) = 10 + 20(-2) = \boxed{-30}$$

$$\boxed{a_4} = 10 + (4-1) \cdot (-2) = 10 + 3(-2) = \boxed{4}$$

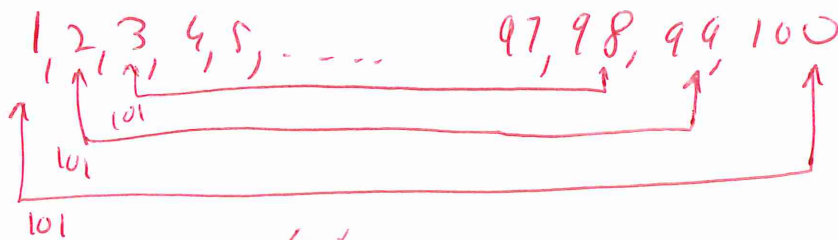
Given two elements, find the sequence:

$$a_3 = 6 \quad a_{10} = 27$$

$$a_{10} = a_3 + (10-3) \cdot d \Rightarrow 27 = 6 + 7 \cdot d$$

$$21 = 7d \Rightarrow \boxed{d = 3}$$

Sum



$$S_{1-100} = \frac{(\text{First} + \text{Last})}{2} \cdot \text{number of elements}$$

$\uparrow$  average

$$= 101 \cdot \frac{100}{2} = \boxed{5050}$$

$$\boxed{S_n = (a_1 + a_n) \cdot \frac{n}{2}}$$

$$a_n = a_1 + (n-1)d$$

$$\boxed{S_n = (2a_1 + (n-1)d) \cdot \frac{n}{2}}$$

Example: Hanukkah candles  $2, 3, 4, \dots, 9 \Rightarrow S_n = \frac{(2+9) \cdot 8}{2} = \boxed{44}$

## Sum (Continued)

$$\sum_{n=1}^{13} (4n+5)$$

① Write down first 3 elements, and last one:

$$a_1 = 4 \cdot 1 + 5 = 9$$

$$a_2 = 4 \cdot 2 + 5 = 13$$

$$a_3 = 4 \cdot 3 + 5 = 17$$

$$\vdots$$
$$a_{13} = 4 \cdot 13 + 5 = 57$$

② Is it arithmetic?  $\rightarrow$  fixed difference,  $d=4$   
YES

③ Calculate sum:

$$S_{13} = (9 + 57) \cdot \frac{13}{2} = \frac{66 \cdot 13}{2} = 33 \cdot 13 = \boxed{429}$$