



Name: _____

Block: _____

Test: Unit 6 and 11

Rational expressions + Polynomials

There are **10 questions** in this quiz. Question number has larger weight.

Standard test time is **40 minutes**.

Four operations calculator is allowed.

Some reminders

- $\frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$
- $\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \cdot \frac{3}{1} = \frac{3}{2}$

===== Start of test

1. Simplify. Remember to note excluded values.

$$\frac{x^2+10x+25}{x^2-9} \cdot \frac{x+3}{x+5}$$

$$\boxed{x \neq 3, -3, -5}$$

$$\frac{(x+5)^2}{(x+3)(x-3)} \cdot \frac{(x+3)}{(x+5)} = \boxed{\frac{x+5}{x-3}}$$

2. Simplify. Remember to note excluded values.

$$(x+3)(x-2) \leftarrow \frac{x^2+x-6}{x^2+6x+9} \cdot \frac{(x+4)^2}{x^2+2x-8}$$

$\downarrow \qquad \qquad \downarrow$
 $(x+3)^2 \qquad (x+4)(x-2)$

$$\boxed{x \neq -3, -4, 2}$$

$$\frac{(x+3)(x-2)}{(x+3)^2} \cdot \frac{(x+4)^2}{(x+4)(x-2)} = \boxed{\frac{x+4}{x+3}}$$

3. Simplify. Remember to note excluded values.

$$\frac{(x+6)(x-6)}{x^2-8x+16} \div \frac{3x-18}{x^2-x-12} \rightarrow 3(x-6)$$

$$\frac{(x-4)^2}{(x-4)(x+3)} = \frac{(x+6)(x-6)}{3(x-4)}$$

$x \neq 4, -3, 6$

$$\frac{(x+6)(x-6)}{(x-4)^2} \cdot \frac{(x-4)(x+3)}{3(x-6)} = \frac{(x+6)(x+3)}{3(x-4)}$$

4. Simplify. Remember to note excluded values.

$$\frac{2x-10}{x^2-25} - \frac{5-x}{25-x^2}$$

$$(x-5)(x+5) \quad (x+5)(5-x)$$

$x \neq 5, -5$

$$\frac{2(x-5)}{(x-5)(x+5)} - \frac{(5-x)}{(5+x)(5-x)} = \frac{2-1}{(x+5)} = \frac{1}{x+5}$$

5. Simplify. Remember to note excluded values.

$$\frac{x^3-8}{x^2-4x+4} - \frac{x^3+3x^2}{x^2+x-6} - \frac{8}{x-2} \rightarrow x^2(x-3)$$

$$(x-2)^2 \quad (x+3)(x-2)$$

$$\boxed{x \neq 2, -3}$$

$$x^3-2^3 = (x-2)(x^2+2x+4)$$

$$\frac{(x-2)(x^2+2x+4)}{(x-2)^2} - \frac{x^2(x-3)}{(x+3)(x-2)} - \frac{8}{(x-2)} =$$

$$= \frac{x^2+2x+4-x^2-8}{(x-2)} = \frac{2x-4}{(x-2)} = \frac{2(x-2)}{(x-2)} = \boxed{2}$$

6. Solve. Show your work!

$$\frac{x-2}{x+2} = \frac{2}{3}$$

$$\boxed{x \neq -2}$$

$$3(x-2) = 2(x+2)$$

$$3x-6 = 2x+4$$

$$\boxed{x = 10}$$

check: $\frac{10-2}{10+2} = \frac{2}{3}$

$$\frac{8}{12} = \frac{2}{3} \checkmark$$

7. Solve. Show your work!

$$\frac{2x+3}{x-1} = \frac{10}{x^2-1} + \frac{2x-3}{x+1}$$

$$(x+1)(x-1)$$

$$\boxed{x \neq 1, -1}$$

$$(2x+3)(x+1) = 10 + (2x-3)(x-1)$$

$$2x^2+5x+3 = 10 + 2x^2-5x+3$$

$$10x = 10 \Rightarrow x = 1$$

But excluded value!

NO solution

8. Divide using long division.

$$(6x^4 - x^3 - 21x^2 + 7x + 5) \div (3x - 5)$$

$$\begin{array}{r}
 2x^3 + 3x^2 - 2x - 1 \\
 3x - 5 \overline{) 6x^4 - x^3 - 21x^2 + 7x + 5} \\
 \underline{6x^4 - 10x^3} \\
 9x^3 - 21x^2 \\
 \underline{9x^3 - 15x^2} \\
 -6x^2 + 7x \\
 \underline{-6x^2 + 10x} \\
 -3x + 5 \\
 \underline{-3x + 5} \\
 0
 \end{array}$$

$$2x^3 + 3x^2 - 2x - 1$$

9. Divide using synthetic division.

$$(6x^4 + 2x^2 - 104) \div (x + 2) = 6x^3 - 12x^2 + 26x - 52$$

$$\begin{array}{c}
 \uparrow \quad \uparrow \\
 +0 \cdot x^3 \quad +0 \cdot x
 \end{array}$$

$$\begin{array}{r|rrrrr}
 -2 & 6 & 0 & 2 & 0 & -104 \\
 & \downarrow & \nearrow -12 & \nearrow +24 & \nearrow -52 & \nearrow 104 \\
 & 6 & -12 & 26 & -52 & 0
 \end{array}$$

$$6x^3 + (-12x^2) + 26x - 52$$

10. Given the expression:

$$9x^2 + x^4 + 4x - 6x^3 - 12$$

a) Write the polynomial in standard form

$$P(x) = \underline{x^4 - 6x^3 + 9x^2 + 4x - 12}$$

b) How many terms are there in $P(x)$?

5

c) What is the degree of the polynomial?

4

d) What is the sign of the leading coefficient?

+

e) What is the end behavior of the graph?

↑ ↑

f) It is given that the polynomial has a root at $x = 3$, and another root at $x = 2$. Find the remaining roots of the polynomial (There's a place to write these on the next page).

(Hint: Start by dividing with the root at $x = 3$)

$$(x-3)(x-2)$$

$\div (x-3) \rightarrow$

$$\begin{array}{r|rrrrr} 3 & 1 & -6 & +9 & +4 & -12 \\ & & 3 & -9 & 0 & 12 \\ \hline & 1 & -3 & 0 & 4 & 0 \end{array}$$

$$(x-3)(x^3 - 3x^2 + 4)$$

$$\begin{array}{r} 1 \\ 0 - x^3 \end{array}$$

$\div (x-2) \rightarrow$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & 0 & 4 \\ & & 2 & -6 & 12 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$$(x-3)(x-2)(x^2 - x - 2)$$

$$(x-2)(x+1)$$

$$P(x) = (x-3)(x-2)^2(x+1)$$

Roots:

1. 3
2. 2
3. 2

4. -1
5. ~~~~~

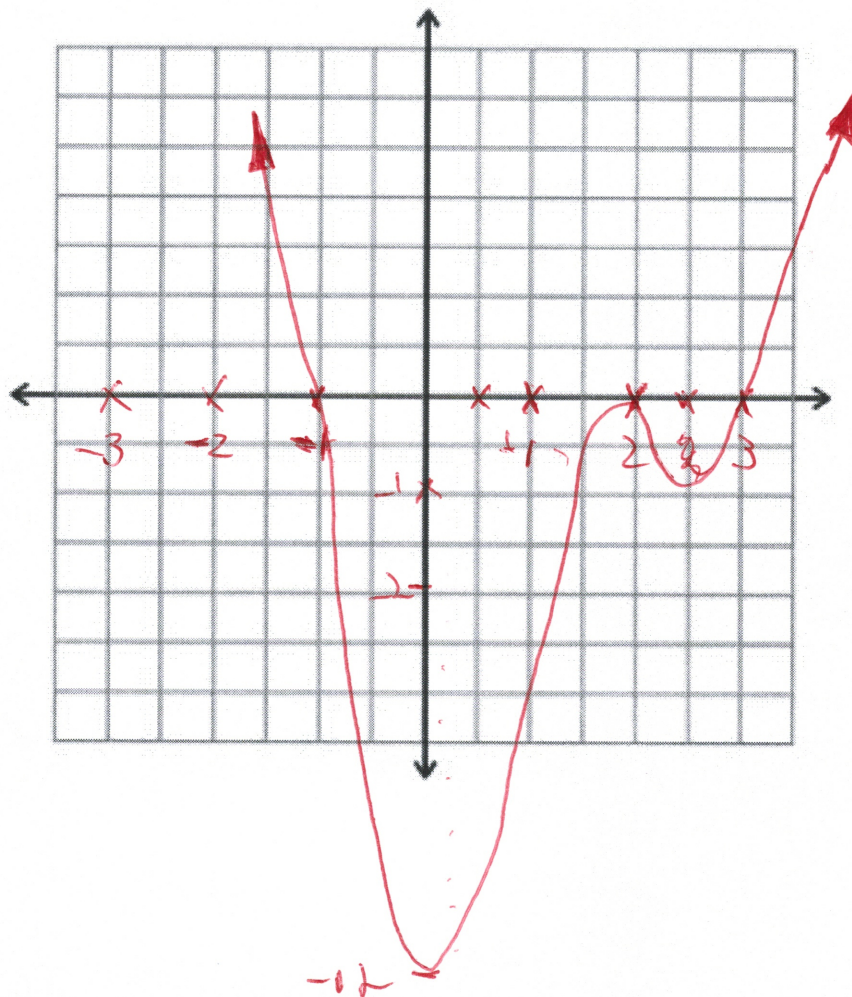
g) Write the polynomial in a factored form.

$$P(x) = \underline{(x-3)(x-2)^2(x+1)}$$

h) What is the y-intercept of the polynomial?

$$\text{y-intercept} = \underline{P(x=0) = -12}$$

i) Utilizing all the information gathered above (and only this information), plot $P(x)$ in the graph below.



This is an extra question for practice. You MAY get a question like this in the test (namely, with complex root).

11. Given the expression:

$$3x(x^2 - x) + x^5 - x^4 + 4 - 4x$$

$3x^3 - 3x^2$

a) Write the polynomial in standard form

$$P(x) = \underline{x^5 - x^4 + 3x^3 - 3x^2 - 4x + 4}$$

b) How many terms are there in $P(x)$?

6

c) What is the degree of the polynomial?

5

d) What is the sign of the leading coefficient?

+

e) What is the end behavior of the graph?

↑↑

f) It is given that the polynomial has a root at $x = 1$, and another root at $x = -2i$. Find the remaining roots of the polynomial (There's a place to write these on the next page).
(Hint: Start by dividing with the root at $x = 1$.)

$\div (x-1)$

$$\begin{array}{r|rrrrrrr} 1 & 1 & -1 & +3 & -3 & -4 & +4 \\ & & 1 & 0 & 3 & 0 & -4 \\ \hline & 1 & 0 & 3 & 0 & -4 & 0 \end{array}$$

$$(x-1)(x^4 + 3x^2 - 4)$$

$$(x-2i)(x+2i) = x^2 + 4$$

$$\begin{array}{r} x^2 + 4 \overline{) x^4 + 3x^2 - 4} \\ \underline{x^4 + 4x^2} \\ -x^2 - 4 \\ \underline{-x^2 - 4} \\ 0 \end{array}$$

$$\begin{aligned} & (x-1)(x-2i)(x+2i)(x+1)(x-1) \\ &= (x-1)^2(x+1)(x^2+4) \end{aligned}$$

Roots:

1. 1
2. $-2i$
3. $+2i$

4. -1
5. $+0$

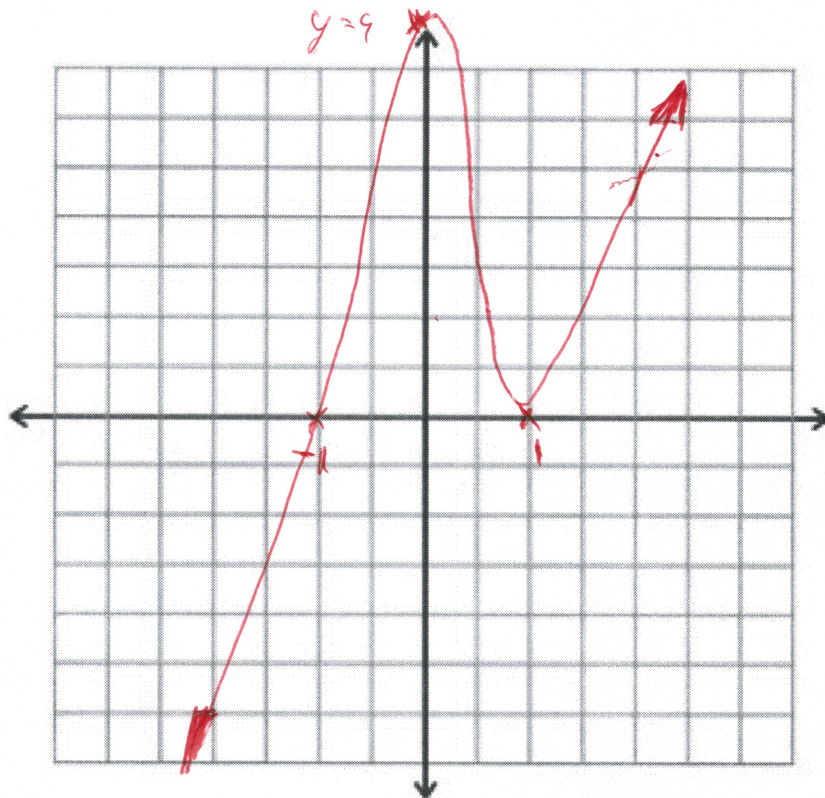
g) Write the polynomial in a factored form.

$$P(x) = \underline{(x-1)^2 (x+1) (x^2+4)}$$

h) What is the y-intercept of the polynomial?

$$\text{y-intercept} = \underline{P(x=0) = 4}$$

i) Utilizing all the information gathered above (and only this information), plot $P(x)$ in the graph below.



==== End of test